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STATIC MIXERS TO PROMOTE AXIAL MIXING

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A new class of static mixers is described. They are intended to homogenize feed streams subject to temporal variations in concentration or temperature. The general design approximates the axial mixing performance of a CSTR without need for moving parts. It does this by creating parallel flow paths with a wide variation in residence times. The residence times are chosen to approximate the exponential distribution of a CSTR. The devices do very little mixing in the radial direction and thus behave like completely segregated stirred tanks. They may be used advantageously in series with a conventional static mixer. Mechanically simple but effective designs are described for both laminar and turbulent flow. Experimental measurements on a four-zone mixer confirm the theoretical design and confirm that the residence time distribution is approximately exponential.

Keywords: mixing; static mixer; motionless mixer; residence time; axial mixing; radial mixing.

INTRODUCTION

Conventional static mixers are designed for radial mixing, and they are quite effective in removing radial variations in composition and temperature. The radial mixing also creates a net velocity profile in the axial direction that approximates piston flow even at low Reynolds numbers. This relatively flat profile is an advantage for many purposes and is often considered a design objective[1]. However, the flat velocity profile is a disadvantage when the input composition or temperature varies with time. There is little dampening of input disturbances since piston flow provides no axial mixing.

The CSTR is the classic device for smoothing fluctuations in composition or temperature. CSTRs and similar recycle systems have an exponential distribution of residence times. They provide good dampening of inlet disturbance in composition or temperature provided the period of the disturbance is less than the mean residence time in the tank. The disadvantage of a CSTR is the need for mechanical circulation.

We describe a new class of motionless mixers designed to promote axial mixing. Specifically, these devices approximate the performance of a CSTR without the need for moving parts. A general criterion is developed for specifying parallel flow paths that produce a nearly exponential distribution of residence times as is characteristic of CSTRs. A mechanically simple device has a first appearance time that is 10% of the mean residence time. The variance of the residence time distribution is similar to that of a CSTR.

Conventional static mixers perform best, relative to an open tube, when the fluid is in deep laminar flow. The new axial mixers work in this regime but also have utility in turbulent flow since turbulence in an open tube increases radial mixing but decreases axial mixing. They do very little mixing in the radial direction and may be used advantageously in series with a conventional static mixer. A series combination of the two types provides both radial and axial mixing, delivering a time-average, spatially homogeneous mixture.

THEORETICAL RESPONSE AND DESIGN CRITERIA

We consider axial mixing in the context of dampening fluctuations in the concentration of some chemical component although the arguments presented will equally apply to the dampening of temperature fluctuations. Suppose a flow stream with time-varying concentration, $C_{in}(t)$, enters a mixer. Then the exit concentration, $C_{out}(t)$, is determined by the residence time distribution of the mixer[2]. Let $f(t)$ denote the differential distribution of residence times. Then for a non-reactive tracer, the outlet response is given by[3]:

$$C_{out}(t) = \int_{-\infty}^{t} C_{in}(\theta) f(t-\theta)\,d\theta = \int_{0}^{\infty} C_{in}(t-\theta) f(\theta)\,d\theta \qquad (1)$$

Consider a sinusoidal input

$$C_{in}(t) = \overline{C}\,[1 + \beta_{in}\sin(\omega t)], \quad \beta_{in} < 1 \qquad (2)$$

where ω is the frequency and $t_p = 2\pi/\omega$ is the period of the sinusoidal disturbance. If this stream is fed to a mixing device, the exit stream will also be sinusoidal with the same period but with a phase shift and attenuation in amplitude, $\beta_{out} \le \beta_{in}$. Specifically,

$$C_{out}(t) = \overline{C}\{1 + \beta_{in}[A\sin(\omega t) - B\cos(\omega t)]\} \qquad (3)$$

where

$$A = \int_0^\infty \cos(\omega\theta) f(\theta)\, \mathrm{d}\theta \quad \text{and} \quad B = \int_0^\infty \sin(\omega\theta) f(\theta)\, \mathrm{d}\theta \tag{4}$$

The outlet concentrations oscillates about $\overline{C}$ with amplitude

$$\beta_{\text{out}} = \beta_{\text{in}} \sqrt{A^2 + B^2} \tag{5}$$

where $0 \le \sqrt{(A^2 + B^2)} \le 1$. The extent of attenuation depends on the frequency of the disturbance, the mean residence time in the mixer, $\bar{t}$, and on the functional form for $f(t)$. The explicit dependence on $\bar{t}$ can be eliminated by using dimensionless time, $\tau = t/\bar{t}$, the dimensionless density function, $f(\tau)$, and dimensionless frequency, $\omega\bar{t}$.

Suppose $f(\tau)$ has the exponential distribution of a CSTR:

$$f(\tau) = \exp(-\tau) \tag{6}$$

Equation (5) gives

$$\beta_{\text{out}} = \frac{\beta_{\text{in}}}{\sqrt{(1 + \omega\bar{t})^2)}} \tag{7}$$

where $\omega\bar{t}$ is the dimensionless frequency of the oscillation. Equation (7) provides a benchmark for the performance of a surge dampening device.

The washout function corresponding to equation (6) is

$$W(\tau) = \int_\tau^\infty f(\tau')\mathrm{d}\tau' = \exp(-\tau) \tag{8}$$

and it is this function that the axial static mixer seeks to approximate.

Figure 1 shows the generic approximation scheme. The flow rate through the mixer is divided between N equally sized elements, and the mean residence time in each element is chosen to match the mean residence time for a portion of the washout function. This gives a series of times, τ_n, $n = 1, 2, \ldots, N$ where

$$\tau_n = \frac{t_n}{\bar{t}} = \frac{N + 1 - n}{N}\left[1 - \ln \frac{N + 1 - n}{N}\right] - \frac{N - n}{N}\left[1 - \ln \frac{N - n}{N}\right] \tag{9}$$

The second group of terms vanishes in the indeterminate case of $n = N$. Table 1 shows results for small N. The entries in this table are values of the mean residence time in the various flow elements normalized by the mean residence time in the mixer as a whole, $\tau_n = t_n/t$.

The effectiveness of the approximation is most easily analyzed when the individual flow elements are assumed to be in piston flow with a sharp distribution of residence times within each element. This assumption is conservative in the sense that there will actually be some spread in residence times within an element, and this spread will improve the performance of the composite mixer. The assumption of piston flow in each element gives the following result for the mixer as a whole:

$$f(\tau) = \sum_{n=1}^{N} \frac{\delta(\tau - \tau_n)}{N} \tag{10}$$

Equation (1) gives

$$C_{\text{out}}(t) = \sum_{n=1}^{N} \frac{C_{\text{in}}(t - t_n)}{N} \tag{11}$$

In words, the average exit concentration at time t is an equally weighted average of the concentrations entering at times $t_1, t_2, \ldots, t_N$ seconds earlier. The equal weighting is a consequence of dividing the total flow equally between the various elements. Equation (5) applies with

$$A = \sum_{n=1}^{N} \frac{\cos(\omega\bar{t})}{N} \quad \text{and} \quad B = \sum_{n=1}^{N} \frac{\sin(\omega\bar{t})}{N} \tag{12}$$

Figure 2 compares the output amplitude, $\beta_{\text{out}}/\beta_{\text{in}}$, for the five-zone approximation to that of a CSTR. The comparison is made at the same value for $\bar{t}$, and the parameter for the comparison is the dimensionless period of the sine wave, $\tau_p = (\omega\bar{t})^{-1}$, shown on the x-axis. If $\tau_p > 1.5$, the performance of the five-zone mixer is nearly identical to that of the conventional CSTR, and if $2 < \tau_p < 4$, the five-zone approximation is actually better than the conventional CSTR. A significant attenuation in noise is possible with relative small mixers.

When $\tau_p < 4$, the behavior of the five-zone approximation is complex. For short periods, signals emerging from two of the flow elements can be in phase, giving little

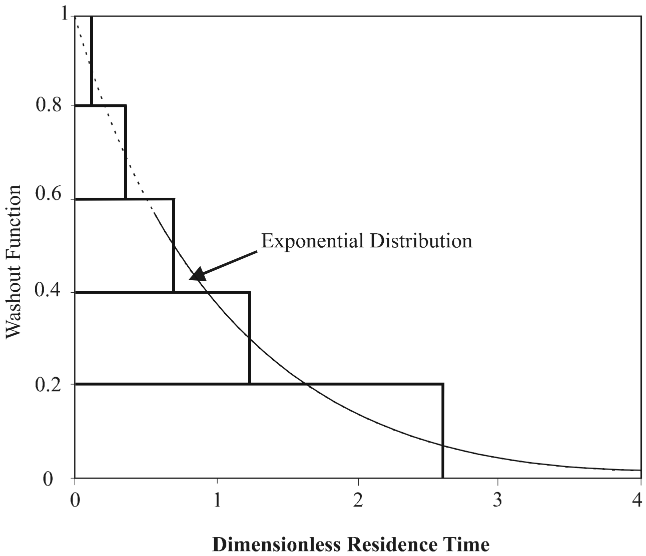

Figure 1. Approximation of an exponential washout function using N flow elements in parallel, $N = 5$ for the illustrated case.

Table 1. Mean residence times in the elements of axial mixers that approximate an exponential distribution of residence times.

	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$	$N = 6$
$n = 1$	1.000	0.307	0.189	0.137	0.107	0.088
$n = 2$		1.693	0.712	0.477	0.360	0.290
$n = 3$			2.099	1.000	0.700	0.542
$n = 4$				2.386	1.223	0.882
$n = 5$					2.609	1.405
$n = 6$						2.792

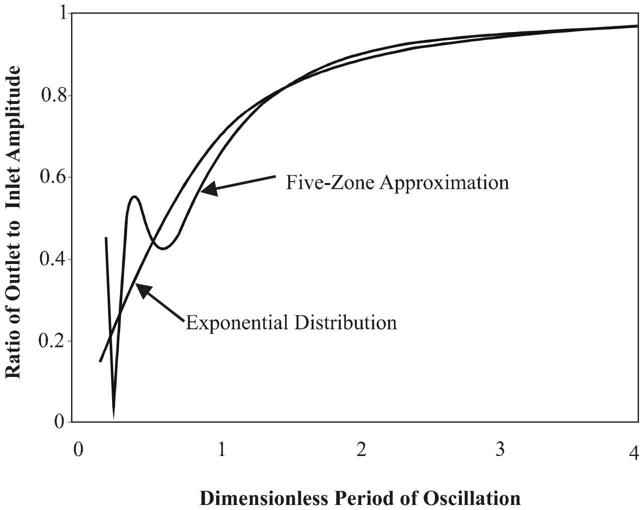

Figure 2. Amplitude response of the five-zone, piston flow approximation to sinusoidal inputs.

attenuation, or out of phase, giving much attenuation. Astatic mixer consisting of two piston flow elements in parallel can completely attenuate a sinusoidal input of fixed frequency. This fact is not of general utility since input noise can have unknown and variable periods. However, it can be useful for eliminating consistent fluctuations such as a 24 h cycle for temperature. Consider a two-element mixer with $t_1 = 0$ and $t_2 = 12$ h. Then, for a sinusoidal input with a 24 h period, the output signals from the two elements will be exactly out of phase, $A = B = 0$, and a sinusoidal input will be completely cancelled. The in-process inventory in the mixer corresponds to just 6 h of throughput. A conventional CSTR with this amount of inventory would have $\bar{t} = 6$, $\omega\bar{t} = 0.25$, and very little dampening. Equation (7) gives $\beta_{out}/\beta_{in} = 0.97$. Of course, should the actual period be 12 h instead of 24, the two output signals will be exactly in-phase, $A = 1$ and $B = 0$, and there will be no attenuation.

Partial phase-matching and mismatching account for the complex behavior shown in Figure 2. The time constants calculated using equation (9) are irrational numbers so that the extreme values $\beta_{out}/\beta_{in} = 0$ and $\beta_{out}/\beta_{in} = 1$ will not occur. Also, the pure time delay of piston flow cannot be achieved physically. Real static mixers will always attenuate noisy signals, but not completely.

Instead of piston flow, suppose the individual elements of the axial mixer have the residence time distribution corresponding to laminar flow in a tube:

$$f(\tau) = 0, \qquad \tau < \frac{\tau_n}{2}$$

$$f(\tau) = \frac{1}{2\tau^3}, \quad \tau > \frac{\tau_n}{2} \tag{13}$$

As before, the mean residence time for each element is chosen to match the mean residence time for a portion of the exponential washout function using equation (9). The residence time density function for the composite system is

$$f(\tau) = 0, \qquad \tau < \tau_1$$

$$f(\tau) = \left(\frac{1}{N}\right)\sum_{i=1}^{n}\frac{\tau_i^2}{2\tau^3}, \quad \frac{\tau_n}{2} < \tau < \frac{\tau_{n+1}}{2} \tag{14}$$

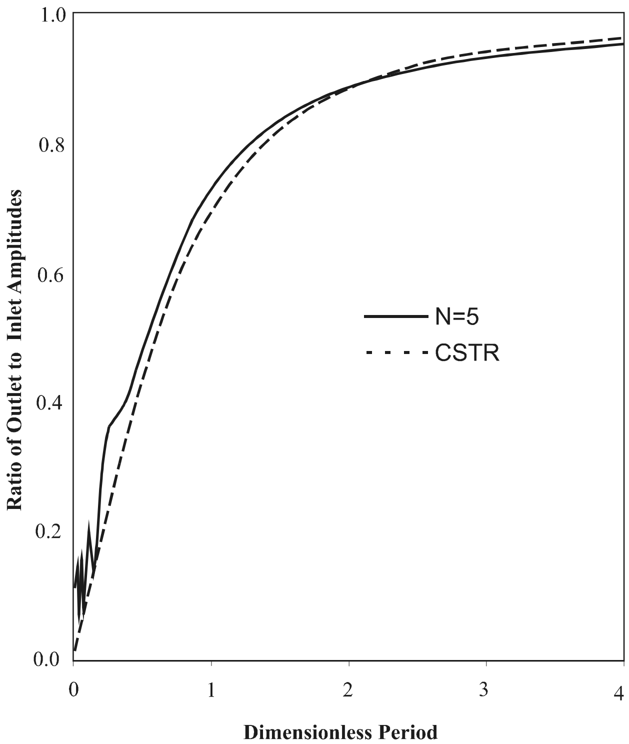

Figure 3. Amplitude response of the five-zone laminar flow approximation to sinusoidal disturbances.

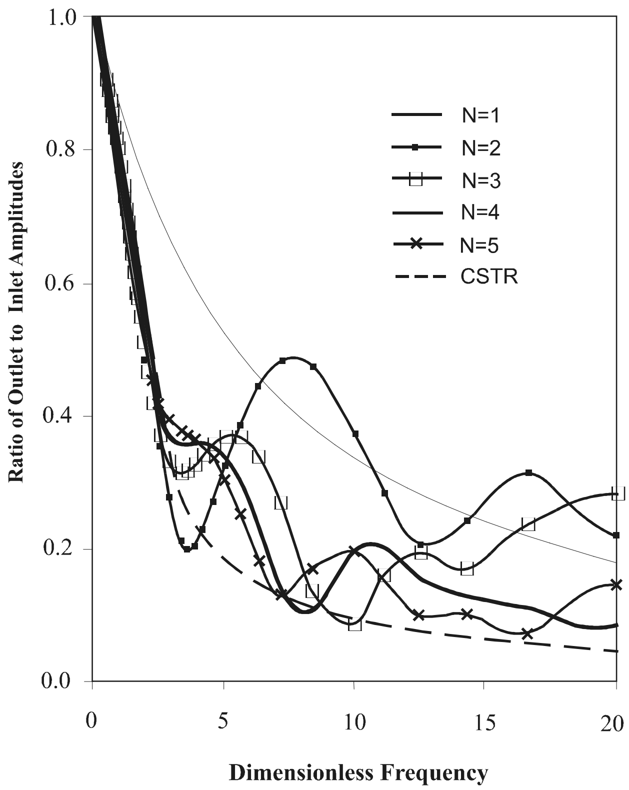

Figure 4. Amplitude response at high frequencies of various laminar flow approximation.

Equations (9) and (11) still apply but the expressions for A and B are now

$$A = \frac{1}{2N} \sum_{n=1}^{N} \tau_n^2 \int_{\tau_{n/2}}^{\infty} \frac{\cos(\omega \bar{t} \tau)\, d\tau}{\tau^3}$$

$$B = \frac{1}{2N} \sum_{n=1}^{N} \tau_n^2 \int_{\tau_{n/2}}^{\infty} \frac{\sin(\omega \bar{t} \tau)\, d\tau}{\tau^3} \quad (15)$$

Figure 3 compares the performance of the laminar flow axial mixers to that of a CSTR. Complex reinforcement and cancellation behavior exits for $\tau_p < 0.15$, but substantial dampening still occurs. Figure 4 shows the high frequency behavior for $N = 1$–5. Even the mixer with $N = 3$ performs reasonably well. High frequency disturbances may be self-dampened through molecular or thermal diffusion. If high-frequency disturbances require additional dampening, an effective means is to use two small (low $\bar{t}$) axial mixers with an intermediate radial mixer.

PHYSICAL IMPLEMENTATION

This section discusses physical designs that can be used for N-zone axial mixers. The following constraints are suitable for most applications:

(1) the mass flow rate must be substantially the same for each value of t_n;
(2) all the flow elements must discharge at a common point;
(3) all the flow elements must have the same pressure drop.

These requirements can be met by a variety of means such as tubes or slits in parallel for laminar flow, tubes with orifice restrictions for turbulent flow, or packed beds. We present two designs suitable for the laminar flow case. Constant viscosity, Newtonian flow is assumed.

Consider a design fabricated from equal-length, circular tubes in parallel. The tube diameter is varied to achieve the desired values for t_n, but the requirement that each zone have substantially the same flow rate means that there must be a much greater number of small diameter tubes than large diameter tubes. The pressure drop down each tube is

$$\frac{dP}{dz} = -\frac{8\mu \bar{u}_n}{R_n^2} \quad (16)$$

where $\bar{u}_n = L/\bar{t}_n$. Table 2 shows design characteristics for a four-zone mixer. The first appearance time for this array is $t_{\min} = t_1/2 = 0.068\bar{t}$.

The four-zone design can be duplicated fairly closely using standard tubing sizes: one tube at 1 inch BWG 13; 12 tubes at 5/8 inch BWG 13; 53 tubes at 3/8 inch BWG 20; and 304 tubes at 1/4 inch BWG 22. However, as shown by the experimental results, precise adherence to the design parameters is unnecessary.

EXPERIMENTAL RESULTS

Figure 5 shows a mixer similar to that specified in Table 2 but with 304 smallest tubes replaced with a packed bed. The Ergun equation suggested that 5/8 inch randomly packed spheres will give the desired value for t_4 when the bed is the same length as the tubes. The entire collection of tubes plus packing fits into a 6 inch pipe. Also, the number of tubes used for zone 3 was changed from 53 to 52 to allow a more symmetrical layout. The experimental mixer was 0.61 m long and had a free volume of 0.0075 m^3. The residence time distribution of this mixer was measured using water as the working fluid and 0.05 g l^{-1} of Congo Red dye as the optical tracer. Nine experimental runs were made at flow rates ranging between 0.5 and 1.5 l $\min^{-1}$. Reynolds numbers based on the open tube diameter ranged from 70 to 210. The Reynolds number for the 1-inch central tube ranged from 85 to 250. Figure 6 shows the entire collection of data. When plotted using the dimensionless residence time, $t/\bar{t}$, there is no discernable trend with respect to flow rate. The washout function has a first appearance time of

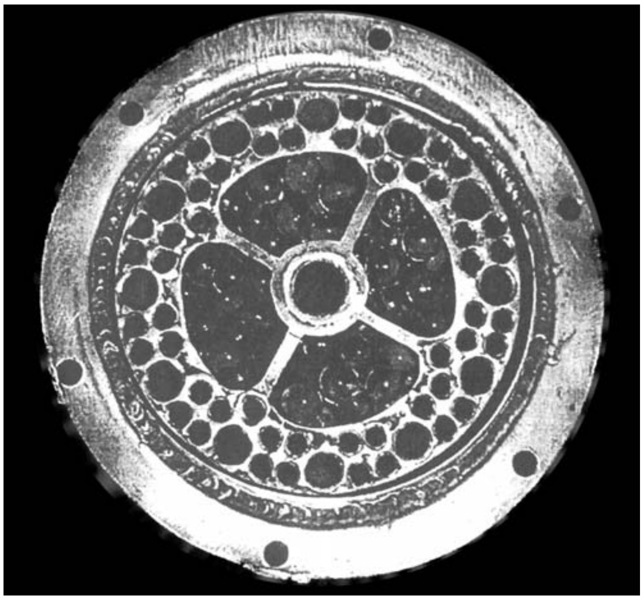

Figure 5. A four-zone axial mixer.

Table 2. Design characteristics for a four-zone axial mixer using circular tubes in the laminar regime.

Zone, n	Mean residence time in zone, $\bar{t}_n$	Relative volume of zone	Relative velocity in zone	Relative radius of tubes in zone	Number of tubes in zone
1	0.137	0.034	7.302	2.702	1
2	0.477	0.119	2.098	1.448	12
3	1.000	0.250	1.000	1.000	53
4	2.386	0.597	0.419	0.647	304
					370

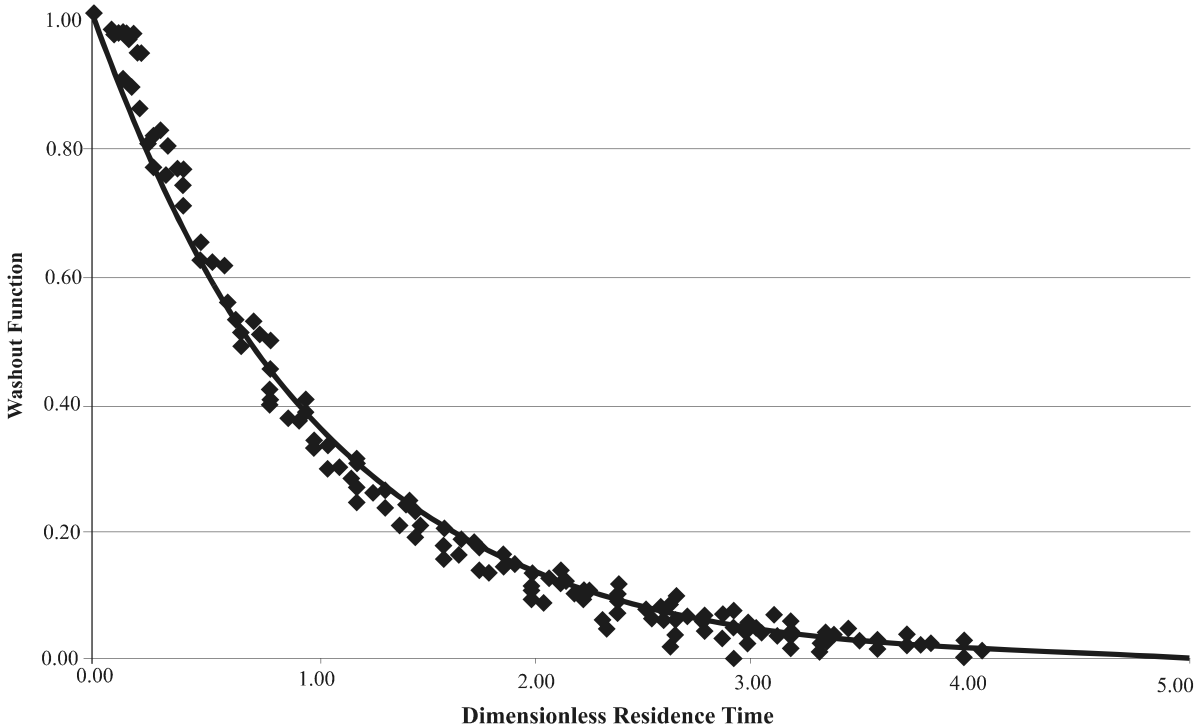

Figure 6. Experimentally measured washout function for the four-zone axial mixer. The smooth curve is the exponential distribution of a CSTR.

about $0.1\bar{t}$ and remains slightly above the exponential distribution until about $0.5\bar{t}$.

CONCLUSIONS

Static mixers designed to attenuate input fluctuations in temperature or composition are technically feasible and relatively simple to construct. When the frequency of the fluctuation is known and constant, as in a daily cycle, a two-zone mixer can provide almost complete attenuation. For unknown or variable frequencies, good overall performance can be achieved by approximating the exponential residence time distribution of a CSTR. A close approximation to the exponential distribution can be achieved with a four-zone mixer constructed using either four sizes of tubing or three sizes of tubing and a packed bed. The theoretical prediction has been confirmed experimentally for a laminar flow system. The dampening performance for the four-zone mixer will be similar to that of a CSTR having the same mean residence time.

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